

Einführung in die Elektrotechnik
 SS2007
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Problem set 12 solutions

■ 1 Diode w/ 2 resistors

We will solve this circuit three ways. First define the givens:

```

VI = 3;
R1 = 150;
R2 = 560;
T = 25;
k = 1.38 * 10-23;
q = 1.6 * 10-19;
VT =  $\frac{k (T + 273)}{q}$ 
0.0257025
  
```

We determine I_0 by using the standard formula:

```

ID = 10 * 10-12;
VD = -0.05;
I0 = Abs [  $\frac{ID}{(\text{Exp}[\frac{VD}{VT}] - 1)}$  ] ;
EngineeringForm [%]
11.7117 * 10-12
  
```

1) Solve by estimate; we assume a diode forward voltage of 0.7 V.

```

I2 =  $\frac{0.7}{R2}$  ;
EngineeringForm [%]
1.25 * 10-3
  
```

2) To solve graphically, we have to plot the load line and the diode characteristic. First find the endpoints of the load line on the voltage axis (Vload) and current axis (Iload):

```

vload = VI (  $\frac{R2}{R1 + R2}$  ) // N
Iload =  $\frac{VI}{R1}$  ;
EngineeringForm [%] // N
2.3662
20. * 10-3
  
```

Plot both on the same axes, using the standard diode formula:

```

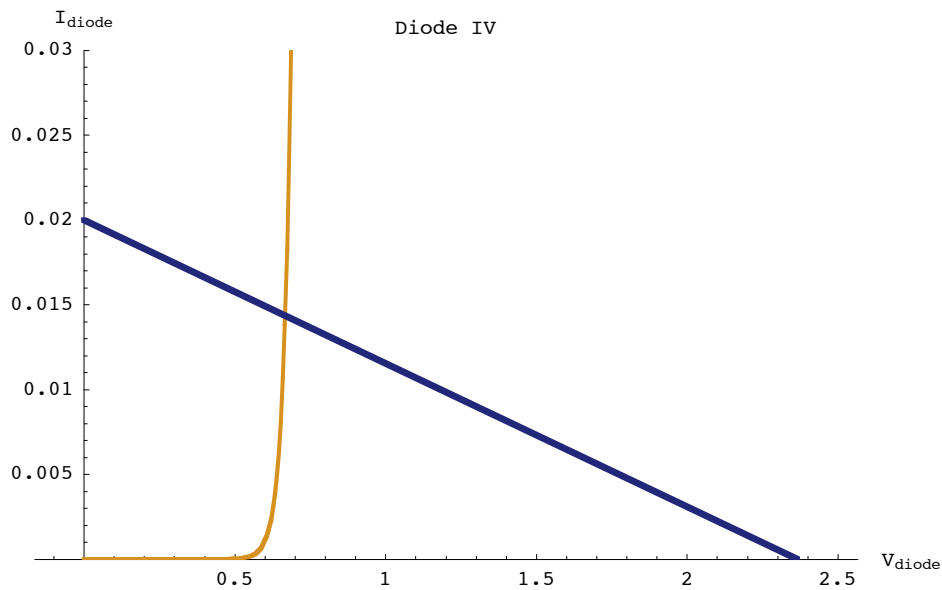
Clear[I_diode];
I_diode[V_] := I0 (Exp[V/VT] - 1)

Plot[{I_diode[V]}, {V, -0.1, 2.5},
  PlotLabel -> "Diode IV",
  AxesLabel -> {"V_diode", "I_diode"},
  PlotRange -> {0, 30 * 10^-3},
  PlotPoints -> 100,
  PlotStyle ->
  {{Thickness[0.005], RGBColor[0.8, 0.5, 0.1]}}]

Show[%, Graphics[{Thickness[0.007], RGBColor[0.1, 0.1, 0.4],
  Line[{{0, Iload}, {Vload, 0}}]}]]]

```

- Graphics -



- Graphics -

Intersection is at about 15 mA.

3) Solve numerically:

```

VD =.; ID =.;
Solve[VI - VD - R1 I0 (Exp[V_D/V_T] - 1) == 0, VD]
{{VD -> 0.54748}}

Solve[VI == R1 ID + VT Log[ID/I0 + 1], ID];
EngineeringForm[%]
{{ID -> 16.3501 * 10^-3}}

```

Graphic & numerical results agree reasonably; estimate with $V_D = 0.7$ V is too low, since real diode voltage (V_D) is only about 0.55 V.

■ 2 Three diodes in series

This exercise is simple, but requires that we use the real exponential characteristic of the diode. Assuming that the diodes are identical and that their voltage drop is identical, we have

$$\begin{aligned} \mathbf{V_{bias}} &= \mathbf{20}; \\ \mathbf{V_{total}} &= \mathbf{2.4}; \\ \mathbf{V_{diode}} &= \frac{\mathbf{V_{total}}}{\mathbf{3}} \\ &= \mathbf{0.8} \end{aligned}$$

Then the diode current which flows is given by (in Amps!)

$$\begin{aligned} \mathbf{V_T} &= \mathbf{26 * 10^{-3}}; \\ \mathbf{I_0} &= \mathbf{100 * 10^{-15}}; \\ \mathbf{I_{diode}} &= \mathbf{I_0 \left(\text{Exp} \left[\frac{\mathbf{V_{diode}}}{\mathbf{V_T}} \right] - 1 \right)} \\ &= \mathbf{2.30625} \end{aligned}$$

Ohm's law then gives us R to give us this current with the given V-drop:

$$\begin{aligned} \mathbf{R} &= \frac{\mathbf{(V_{bias} - V_{diode})}}{\mathbf{I_{diode}}} \\ &= \mathbf{8.32519} \end{aligned}$$

Thus low R and high I are required to even reach $V_{diode} = 0.8$ V.

■ 3 Emitter follower

This circuit can easily be solved by hand. We know β , R_B , R_C , and R_E ; $V_{DD} = 5$. Let's assume a forward diode voltage of $V_D = 0.7$ V.

$$\begin{aligned} \mathbf{\beta} &= \mathbf{90}; \\ \mathbf{R_B} &= \mathbf{47 * 10^3}; \\ \mathbf{R_C} &= \mathbf{1.5 * 10^3}; \\ \mathbf{R_E} &= \mathbf{220}; \\ \mathbf{V_{DD}} &= \mathbf{5}; \\ \mathbf{V_D} &= \mathbf{0.7}; \end{aligned}$$

a) With an operating point of $V_o = 2.5$, we have

$$I_C R_C = V_{DD} - 2.5 \text{ or } I_C = \frac{V_{DD} - 2.5}{R_C}$$

The $I_B = \frac{I_C}{\beta}$ and the input voltage is:

$$V_i = I_B R_B + (\beta + 1) R_E I_B + V_D.$$

Calculating numerically (I_C in mA, I_B in μ A, V_i in Volts);

```

Ic =  $\frac{V_{DD} - 2.5}{R_C}$  ;
EngineeringForm[103 * %]
Ib =  $\frac{I_C}{\beta}$  ;
EngineeringForm[106 * %]
Vi(op point) = Ib Rb + (β + 1) RE Ib + Vd
1.66667
18.5185
1.94111

```

b) The maximum input swing is defined by the maximum swing of the output, namely 0 to V_{DD} .

Maximum V_o is V_{DD} , which occurs for $I_C = 0$ so $I_B = 0$ and thus for an input voltage below the diode forward bias, thus

$$V_{in(min)} \leq 0.7 V$$

Minimum V_o is when the bipolar is cutoff, so $V_{CE} = 0$. At that point,

$$V_o = I_C R_E \left(\frac{\beta+1}{\beta}\right) \text{ but also } V_o = V_{DD} - I_C R_C.$$

We can thus write

$$I_C R_E \left(\frac{\beta+1}{\beta}\right) = V_{DD} - I_C R_C \text{ or } I_C = \frac{V_{DD}}{R_E \left(\frac{\beta+1}{\beta}\right) + R_C} \text{ such that of course } I_B = \frac{I_C}{\beta}.$$

Then as before:

$$V_{i(max)} = I_B R_B + (\beta + 1) R_E I_B + V_D$$

Numerically (I_C in mA, I_B in μA , V_i in Volts):

```

Ic =  $\frac{V_{DD}}{R_E \left(\frac{\beta+1}{\beta}\right) + R_C}$  ;
EngineeringForm[103 * %]
Ib =  $\frac{I_C}{\beta}$  ;
EngineeringForm[106 * %]
Vi(max) = Ib Rb + (β + 1) RE Ib + Vd
Null
2.90285
32.2539
2.86166

```

So the voltage swing before the transistor saturates or cuts off is $0.7 \leq U_i \leq 2.86 V$.

c) Amplification: since $V_i = I_B R_B + (\beta + 1) R_E I_B + V_D$ then the change in V_i is expressed as

$$\Delta V_i = \Delta I_B R_B + (\beta + 1) R_E \Delta I_B = \Delta I_B (R_B + (\beta + 1) R_E).$$

Likewise, at the output: $V_o = V_{DD} - I_C R_C$ so that $\Delta V_o = -\Delta I_C R_C = -\beta \Delta I_B R_C$.

$$\text{Replacing } \Delta I_B \text{ with } \Delta I_B = \frac{\Delta V_i}{(R_B + (\beta + 1) R_E)}, \text{ we have } \Delta V_o = -\beta R_C \frac{\Delta V_i}{(R_B + (\beta + 1) R_E)}$$

The amplification is thus

$$A = \frac{\Delta V_o}{\Delta V_i} = - \frac{\beta R_C}{(R_B + (\beta + 1) R_E)}$$

Numerically:

$$\mathbf{A} = - \frac{\beta R_C}{(R_B + (\beta + 1) R_E)}$$

-2.01432