Micro-optics

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Exercise 1

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1 Maxwell equations (60 P)

The Maxwell equations provide information in a wide sense about sources and vorticity of electromagnetic fields. $\vec{F}$ denotes an arbitrary vector field.

1.1 Sources and curl (5 P)

When do we say $\vec{F}$ is solenoidal and when can we say it is irrotational?

1.2 Fields (16 P)

Investigate the following 4 fields with regard to source strength and vorticity. First, sketch the fields at the cirumference of the unit circle (center at point of origin) in the x-y-plane or another suitable plane. Employ e.g. the relative scale $\left|\frac{\vec{F}}{F}\right| = 1cm$.

I. $\vec{F} = F\left(\vec{e}_x + 2\vec{e}_z\right)$

II. $\vec{F} = F\,\frac{x\vec{e}_x + y\vec{e}_y + z\vec{e}_z}{\sqrt{x^2+y^2+z^2}}$

III. $\vec{F} = F\,\frac{y\vec{e}_x - x\vec{e}_y}{\sqrt{x^2+y^2}}$

IV. $\vec{F} = F\,\frac{(x-y)\vec{e}_x + (x+y)\vec{e}_y}{\sqrt{x^2+y^2}}$

1.3 Source strength and vorticity (20 P)

Calculate the source strengths and vorticities respectively.

1.4 Interpretation (9 P)

What do the results qualitatively say? Give some practical examples from electrodynamics/-statics or from fluidics.

1.5 Special example (10 P)

Another more involved example is $\vec{F} = -Fx^2\vec{e}_y$.

Calculate the curl of this field. In this case the curl or rotation is not directly seen when drawing the field. How can the rotation be plausibly explained and what can be said about the direction of rotation?

2 Wave equation (40 P)

2.1 Conventions (6 P)

In most optics textbooks, the field quantities $\vec{E}, \vec{H}$, etc. are complex because calculations using exponentials are much easier than calculations involving trigonometric functions.

As an engineer you may want to know the actual electric field of a light wave in Volts per meter. How do you calculate it from the complex field quantities?

2.2 Planar wave (14 P)

Show that the scalar wave $U(z,t) = U_0 \cdot \exp\left[j(\vec{k}\cdot\vec{r}) - j\omega t\right]$ is a solution of the wave equation in Cartesian coordinates! (Hint: Recall that $\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$)

2.3 Spherical wave (20 P)

For a scalar field in spherical polar coordinates, i.e., $U = U(r,\theta,\phi)$, the Laplacian is given by:

$$\Delta U = \frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial \phi^2} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{1}{r^2}\cot\theta\frac{\partial U}{\partial \theta} \tag{1}$$

Using this expression, write down the wave equation for a scalar field $U = U(r)$ and show that

$$U(r,t) = \frac{U_0}{r}\exp\left[jkr - j\omega t\right] \tag{2}$$

is a solution of it. Why is no scalar product $\vec{k}\cdot\vec{r}$ required in the expression for the spherical wave?