

Micro-optics

Winter semester 07/08

Exercise 2

Prof. Hans Zappe

Tutors: Dr. W. Mönch, Dr. A. Seifert

Discussion sections: Nov 7 & 8, 2007

1 Polarization (40 P)

1.1 Polarization states (20 P)

Given a wave with electric field components

$$\vec{E} = E_x \sin(\omega t - kz + \delta_x) \vec{e}_x + E_y \sin(\omega t - kz + \delta_y) \vec{e}_y \quad (1)$$

calculate the polarization state for the following 4 cases:

- I. $E_x = -E_y$, $\delta_x = -\pi/2$, $\delta_y = +\pi/2$
- II. $E_x = E_y$, $\delta_x = 0$, $\delta_y = +\pi/2$
- III. $E_x = E_y$, $\delta_x = 0$, $\delta_y = -\pi/2$
- IV. $E_x = E_y$, $\delta_x = 0$, $\delta_y = -\pi/4$

1.2 Elliptical polarization (20 P)

Show that the wave which results when we superimpose two linearly polarized waves,

$$E_x = E_{x0} \sin(\omega t - kz) \quad (2)$$

$$E_y = E_{y0} \sin(\omega t - kz + \delta) \quad (3)$$

is given by an equation which defines an ellipse.

2 Gaussian beams (30 P)

Gaussian beams are realistic optical waves which only in special cases become planar or spherical.

2.1 Ideal cases (5 P)

In what part of the Gaussian beam can it be well-approximated by a plane wave or a spherical wave (\rightarrow sketch)?

2.2 Wave equation (10 P)

Does the mathematical description of a Gaussian beam satisfy the wave equation? Which assumptions or approximations have to be made such that the 'Gaussian beam' solves the wave equation?

2.3 Example (15 P)

A 1 mW HeNe laser produces a Gaussian beam of wavelength $\lambda = 633$ nm and a spot size $2W_0 = 0.1$ mm. Determine the angular divergence θ_0 of the beam, its depth of focus $2z_0$ and its diameter at $z = 3.5 \times 10^5$ km (\cong the distance to the moon).

3 Intensity (30 P)

3.1 Spherical wave (7 P)

Write down an expression for a spherical wave. How does the intensity of a spherical wave depend on the distance from its center?

3.2 Poynting vector (15 P)

The Poynting vector \vec{S} is defined as $\vec{S} = \vec{E} \times \vec{H}$, and the intensity I of a wave is defined as the temporal average of \vec{S} . Find an expression for I for a planar wave traveling in a medium with a refractive index n . What are the units of S and I , and how may they be interpreted?

3.3 Particle picture of light (8 P)

As a complementary description of light, the particle picture was developed in the framework of quantum mechanics. The basic idea is that light occurs as discrete particles called photons with an energy of $E = h\nu$ (h is Planck's constant, and ν the frequency of the light) such that the intensity may be expressed as $I = h\nu\Phi$.

Given that Φ is the photon (particle) flux, i.e. the number of photons per unit area and per unit time crossing a reference area (e.g., a photodiode),

- Estimate Φ for a red ($\lambda = 633$ nm) Helium-Neon laser with a power of 23 mW and a beam cross sectional area of 1 mm².
- Estimate I for the flux of footballs measured by the goalkeeper of the SC Freiburg in his goal.

