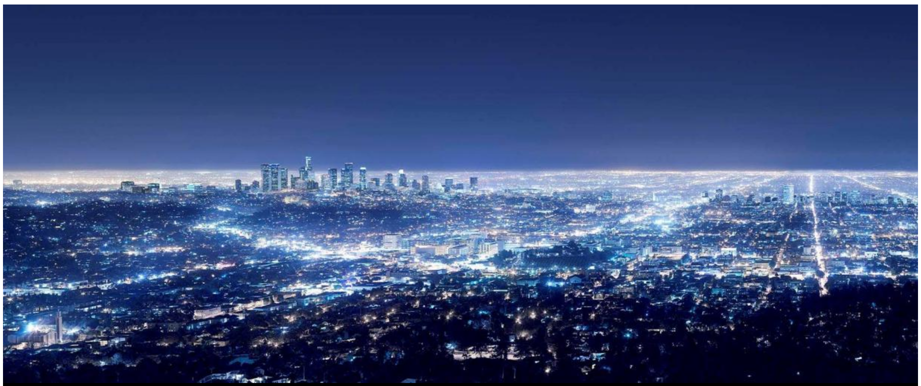


Rüdiger Franke, ABB AG

Huge Quadratic Programming
Embedded QP Workshop
Freiburg, March 19, 2014

Contents
More features that are unique in their combination

- Initial development
- Module structure
- (S)QP algorithm
- Applications overview
- Application in ABB Dynamic Optimization
- Example: NMPC for power plant start-up

Huge Quadratic Programming
Initial development

1994: development of solver HQP started

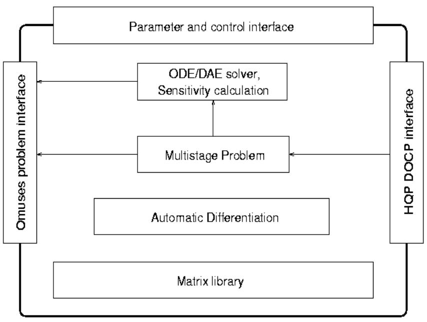
- Goal: apply interior point methods to solution of large-scale QP problems arising in optimal control (S.J. Wright, 1993)
- based on open source software (Meschach)
- designed as C++ framework with command interface (Tcl)
- extension to SQP

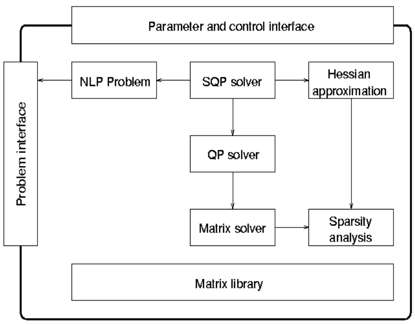
1997: development of front-end Omuses started

- Goal: simplify formulation of optimal control problems
- Solution of differential and sensitivity equations
- Automatic differentiation (ADOL-C)
- External model interfaces (S-function, FMI)

HQP is open source, see: https://github.com/omuses/hqp

Module structure of HQP / Omuses framework





Omuses:

- multi-stage front-end
- convert Dynamic Optimization problem to NLP
- solve DAEs, including sensitivities (e.g. using DASPK)
- interface to Simulink S-function (DLL)

HQP:

- sparse SQP solver
- Interior Point QP solver
- uses Meschach matrix library
- uses Tcl
- C++ framework

Dynamic optimization problem

$$J = f_f[x(t_f), y(t_f)] + \int_{t=0}^{t_f} f_0[t, x(t), u(t), y(t)]\, dt \longrightarrow \min_{u(t), x(0)}$$

$$\begin{aligned} &s.t. \\ &F[t, x(t), \dot{x}(t), u(t), y(t), p] = 0, \quad I[x(0), \dot{x}(0), u(0), y(0), p] = 0, \\ &g[t, x(t), u(t), y(t), p] \geq 0, \\ &g_f[x(t_f), y(t_f), p] \geq 0 \end{aligned}$$

For dynamic system model and optimization time horizon $[0, t_f]$

- find control $u(t)$ (and/or initial states $x(0)$) that minimize criterion J
- subject to differential-algebraic model equations F and initial conditions I
- and further constraints g

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Treatment by HQP optimization solver
Multi-stage Control Vector Parameterization

- parameterize control $u(t) = u(u^k)$, $k=0,1,\ldots,K-1$
- convert dynamic optimization problem to discrete-time optimal control problem:

$$J = f_0(x^K) + \sum_{k=0}^{K-1} f_0(x^k, u^k) \longrightarrow \min_{x^0, u^k}$$

$$\begin{aligned} &s.t. \\ &x^{k+1} = f^k(x^k, u^k), \quad k = 0, \ldots, K-1 \\ &c^k(x^k, u^k) \geq 0, \quad k = 0, \ldots, K-1 \\ &c^K(x^K) \geq 0 \end{aligned}$$

- solve as large-scale nonlinear programming problem with vector of optimization variables v:

$$v = \begin{pmatrix} x^0 \\ u^0 \\ x^1 \\ u^1 \\ \vdots \\ x^{K-1} \\ u^{K-1} \\ x^K \end{pmatrix}$$

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HQP
Lagrange-Newton type SQP algorithm

Large-scale nonlinear programming problem

$$J(\mathbf{x}) \to \min_{\mathbf{x}}, \quad J : \mathbb{R}^n \to \mathbb{R}^1$$

$$\text{s.t.} \quad \mathbf{h}(\mathbf{x}) = \mathbf{0}, \quad \mathbf{h} : \mathbb{R}^n \to \mathbb{R}^{m_e},$$

$$\mathbf{g}(\mathbf{x}) \geq \mathbf{0}, \quad \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^m.$$

Apply quasi-Newton method to Lagrangian function

$$L(\mathbf{x}, \mathbf{y}, \mathbf{z}) = J(\mathbf{x}) - \mathbf{y}^T \mathbf{h}(\mathbf{x}) - \mathbf{z}^T \mathbf{g}(\mathbf{x})$$

During each iteration solve linear-quadratic appoximation

$$\frac{1}{2}\mathbf{s}^T \mathbf{Q}_i \mathbf{s} \; + \; \left(\nabla_{\mathbf{x}} J(\mathbf{x}^i)\right)^T \mathbf{s} \; \to \; \min_{\mathbf{s}}, \text{ so that}$$

$$\nabla_{\mathbf{x}} \mathbf{h}(\mathbf{x}^i)\mathbf{s} \; + \; \mathbf{h}(\mathbf{x}^i) = \mathbf{0}, \quad \nabla_{\mathbf{x}} \mathbf{g}(\mathbf{x}^i)\mathbf{s} \; + \; \mathbf{g}(\mathbf{x}^i) \geq \mathbf{0}$$

giving QP

$$\min_{\mathbf{s}} \left\{ \frac{1}{2}\mathbf{s}^T \mathbf{Q}\mathbf{s} + \mathbf{c} \mid \mathbf{A}\mathbf{s} + \mathbf{b} = \mathbf{0}, \; \mathbf{C}\mathbf{s} + \mathbf{d} \geq \mathbf{0} \right\}$$

($\mathbf{Q}$ symmetric, positive definite; $\mathbf{A}$ and $\mathbf{C}$ of full rank)

HQP
Hessian approximation – preferrably multi-rank BFGS

Partially separable Lagrangian – block-diagonal Hessian
(H.-G. Bock, K.J. Plitt, 1984):

$$L(\mathbf{x}, \mathbf{y}, \mathbf{z}) = L^K(\mathbf{x}^K, \mathbf{z}^K) \; + \; \sum_{k=0}^{K-1} \left[L^k(\mathbf{x}^k, \mathbf{u}^k, \mathbf{y}^k, \mathbf{z}^k) + (\mathbf{y}^k)^T \mathbf{x}^{k+1} \right]$$

$$\nabla^2_{\mathbf{xx}} L = \begin{pmatrix} \nabla^2_{\mathbf{x}^0\mathbf{x}^0} L & \nabla^2_{\mathbf{x}^0\mathbf{u}^0} L & & & & \\ \nabla^2_{\mathbf{u}^0\mathbf{x}^0} L & \nabla^2_{\mathbf{u}^0\mathbf{u}^0} L & & & & \\ & & \ddots & & & \\ & & & \nabla^2_{\mathbf{x}^{K-1}\mathbf{x}^{K-1}} L & \nabla^2_{\mathbf{x}^{K-1}\mathbf{u}^{K-1}} L & \\ & & & \nabla^2_{\mathbf{u}^{K-1}\mathbf{x}^{K-1}} L & \nabla^2_{\mathbf{u}^{K-1}\mathbf{u}^{K-1}} L & \\ & & & & & \nabla^2_{\mathbf{x}^K\mathbf{x}^K} L \end{pmatrix}$$

HQP
Interior point QP solver

Extended criterion

$$\frac{1}{2}\mathbf{s}^T\mathbf{Q}\mathbf{s} + \mathbf{c}^T\mathbf{s} - \mu\sum_{i=1}^{m}\ln w_i \rightarrow \min_{\mathbf{s}}$$

$$\text{where} \quad \mathbf{w} = \mathbf{C}\mathbf{s} + \mathbf{d}, \quad \mu > 0$$

Extended Karush-Kuhn-Tucker conditions

$$\begin{aligned}
\mathbf{Q}\mathbf{s} + \mathbf{c} - \mathbf{A}^T\mathbf{y} - \mathbf{C}^T\mathbf{z} &= \mathbf{0},\\
\mathbf{A}\mathbf{s} + \mathbf{b} &= \mathbf{0},\\
\mathbf{w} = \mathbf{C}\mathbf{s} + \mathbf{d} &> \mathbf{0},\\
\mathbf{z} &> \mathbf{0},\\
\mathbf{Z}\mathbf{W}\mathbf{e} - \mu\mathbf{e} &= \mathbf{0},
\end{aligned}$$

Apply Newton's method to solve KKT for $\mu \rightarrow 0$.

HQP
Matrix solution approaches

Symmetric, indefinite system (SpBKP)

$$\begin{pmatrix} -\mathbf{Q} & \mathbf{A}^T & \mathbf{C}^T \\ \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \mathbf{Z}_j^{-1}\mathbf{W}_j \end{pmatrix} \begin{pmatrix} \delta\mathbf{s} \\ \delta\mathbf{y} \\ \delta\mathbf{z} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{W}_j\mathbf{e} - \mu_j\mathbf{Z}_j^{-1}\mathbf{e} \end{pmatrix}$$

Reduced system (RedSpBKP)

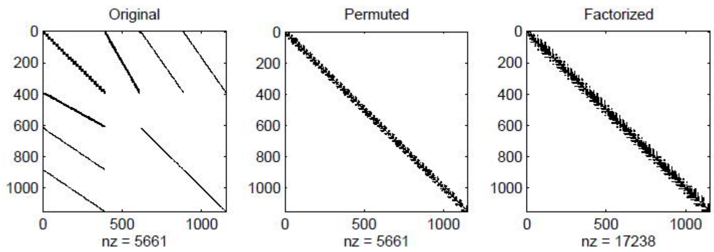
$$\begin{pmatrix} -\mathbf{Q} - \mathbf{C}^T\mathbf{W}_j^{-1}\mathbf{Z}_j\mathbf{C} & \mathbf{A}^T \\ \mathbf{A} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \delta\mathbf{s} \\ \delta\mathbf{y} \end{pmatrix} = \begin{pmatrix} -\mathbf{C}^T\mathbf{W}_j^{-1}\mathbf{Z}_j\mathbf{r}_3^j \\ \mathbf{0} \end{pmatrix}$$

$$\begin{aligned}
\delta\mathbf{z} &= \mathbf{W}_j^{-1}\mathbf{Z}_j(\mathbf{r}_3^j - \mathbf{C}\delta\mathbf{s})\\
\mathbf{r}_3^j &= \mathbf{W}_j\mathbf{e} - \mu_j\mathbf{Z}_j^{-1}\mathbf{e}
\end{aligned}$$

Block factorization method (LQDOCP)

See E. Arnold, 1987

HQP
Examplary spy plots (Zambezi example, 24 time steps)



Huge Quadratic Programming
Research Applications

- Wind turbine control
- Batch process control
- Energy systems
- Water systems

See

- D. Schlipf, D.J. Schlipf, M. Kühn: Nonlinear model predictive control of wind turbines using LIDAR. *Wind Energy*, 16(7):1107 – 1129, 2013.
- Z.K. Nagy, B. Mahn, R. Franke, and F. Allgöwer. Evaluation study of an efficient output feedback nonlinear model predictive control for temperature tracking in an industrial batch reactor. *Control Engineering Practice*, 15(7):839 – 850, 2007.
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- G. Reichl. *Optimierte Bewirtschaftung von Kläranlagen basierend auf der Modellierung mit Modelica.* Dissertation, Cuvillier Verlag, Göttingen, 2005.
- R. Franke. Integrierte dynamische Modellierung und Optimierung von Systemen mit saisonaler Wärmespeicherung, Dissertation, volume 394 of *Fortschritt-Berichte VDI*, series 6. VDI-Verlag, Düsseldorf, 1998.

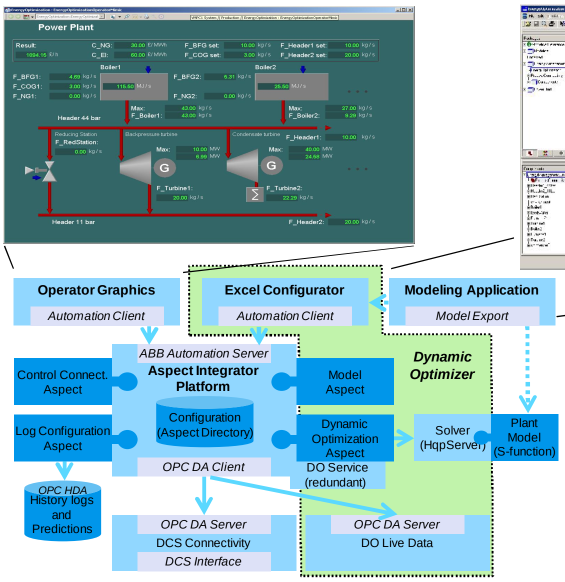
Huge Quadratic Programming
Running round the clock all over the world

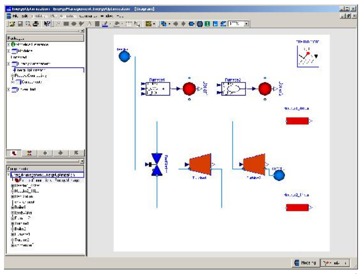
- Energy management of virtual power plants
- Water management in large canal systems
- Anti-sway control of boom cranes
- Online optimization of power generation units

See

- R. Franke, HJ. Schönung, M. Blaumann, A. Frick, and S. Kautsch: Real-time and Intraday Optimization of Multiple Power Generation Units. *PowerGen Europe*, 2013.
- J. Wagenpfeil, E. Arnold, H. Linke, and O. Sawodny: Modeling and optimized water management of artificial inland waterway systems. *Journal of Hydroinformatics*, 15(2):348–365, 2013.
- J. Neupert, E. Arnold, O. Sawodny, and K. Schneider: Tracking and anti-sway control for boom cranes. *Control Engineering Practice*, 18(1):31–44, 2010.
- R. Franke and L. Vogelbacher. Nonlinear model predictive control for cost optimal startup of steam power plants. *at – Automatisierungstechnik*, 54(12):630–637, 2006.
- H. Linke, E. Arnold, and R. Franke. Optimal water management of a canal system. In J.C. Refsgaard and E.A. Karalis, editors, *Operational Water Management*, pages 211–218. Balkema, Rotterdam, 1997.

Advanced Control with ABB Dynamic Optimization





$$J = -\sum_{Boiler_i, i=1,\ldots,n} c_{NG,i}\dot{Q}_{NG,i} + \sum_{Turbine_j, j=1,\ldots,m} c_{El,j}P_{El,j} \longrightarrow \max_{\dot{Q}_{NG,i},\dot{Q}_{BFG,i},\dot{Q}_{COG,i},\dot{Q}_{Oil,i},P_{El,j},\dot{m}_{RedStations}}$$

s.t.

$$\dot{Q}_{BFG} = \dot{Q}_{BFG,provided}, \quad \dot{Q}_{COG} = \dot{Q}_{COG,provided}$$

$$\dot{m}_{44bar} = \dot{m}_{44bar,demand}, \quad \dot{m}_{11bar} = \dot{m}_{11bar,demand}$$

$$0 = F(x), \quad F \equiv \text{Process model with balances}$$

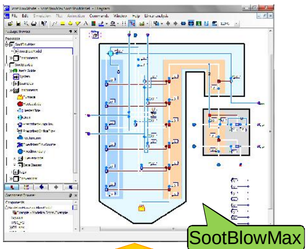
$$\dot{m}_{Boiler_i,\min} \le \dot{m}_{Boiler_i} \le \dot{m}_{Boiler_i,\max}, \quad i = 1,\ldots,n$$

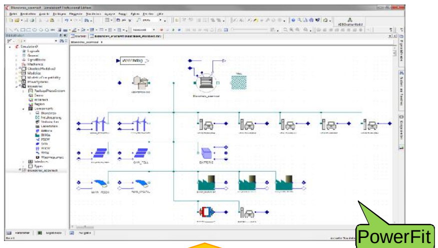
$$P_{Turbine_j,\min} \le P_{Turbine_j} \le P_{Turbine_j,\max}, \quad j = 1,\ldots,m$$

$$\ldots$$

ABB OPTIMAX® Dynamic Optimization using HQP
Standardized application engineering and deployment

Modelica for application engineering (www.modelica.org)

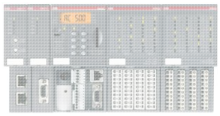




FMI (Functional Model Interface) for deployment (www.fmi-standard.org)

Server-PC

Industrial-PC



Conceptionally: Controller

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NMPC for boiler start-up optimization
Motivation

- Goal and customer benefit
  - reduced operating cost in de-regulated energy market
  - Reduced cost for start-up (> 10%)
  - payback time < 2 years
- Applied Technology
  - Nonlinear Model Predictive Control (NMPC)

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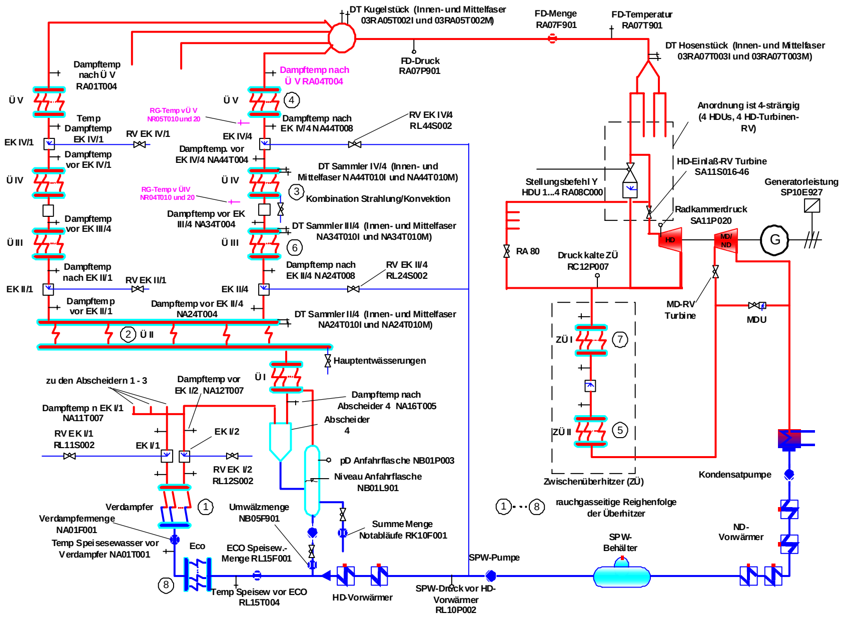
Boiler start-up optimization – process model overview



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Numerical details of online optimization program

- Boiler Model
  - Object-oriented physical model with 68 components
  - Mathematically: High-index DAE with 1229 variables
  - Non-linear due to water-steam properties and heat transfer between steam and metal
- Optimization problem
  - 90 sampling intervals (prediction over 90 minutes with one minute sampling) (meaning 1229x91 = **111839 variables treated during optimization**)
  - Need appropriate reduction of problem formulation
  - Actual size of optimization problem solved on-line:
    - **2902 optimization variables**
    - **1281 equality constraints (from model equations)**
    - **4072 inequality constraints (thermal stress and further process constraints)**
- Solution times for on-line application in real-time
  - initial solution took about 5 minutes immediately before startup
  - then update every minute -- **about 40 seconds solution time per update**
  - Update is required to reject disturbances, e.g. burner failure or overshoot, and to account for model errors

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Dynamic optimization program

$$J = \int_{t=0}^{t_f} \frac{[T(t)-T_{set}]^2}{w_T^2} + \frac{[p(t)-p_{set}]^2}{w_p^2} + \frac{[q_m(t)-q_{m,set}]^2}{w_{q_m}^2}\, dt \longrightarrow \min_{q_F(t), Y_{HPB}(t)}$$

$s.t.$

$\dot{x}(t) = f[x(t), q_F(t), Y_{HPB}(t), Y_{AW}(t)], \quad x(0) = \bar{x}$ ➢ **Process model**

$q_{F,\min} \le q_F \le q_{F,\max}, \quad \dot{q}_{F,\min} \le \dot{q}_F \le \dot{q}_{F,\max}$ ➢ **bounds on fuel**

$0 \le Y_{HPB}(t) \le 1$ ➢ **bounds on valve positions**

$\Delta T_{\min,i} \le \Delta T_i(t) \le \Delta T_{\max,i}, \quad i = 1,\ldots,n$ ➢ **thermal stresses**

General startup optimization problem:

Optimal transition to new operating point (T,p,q_m) subject to control bounds and state constraints (esp. thermal stresses).

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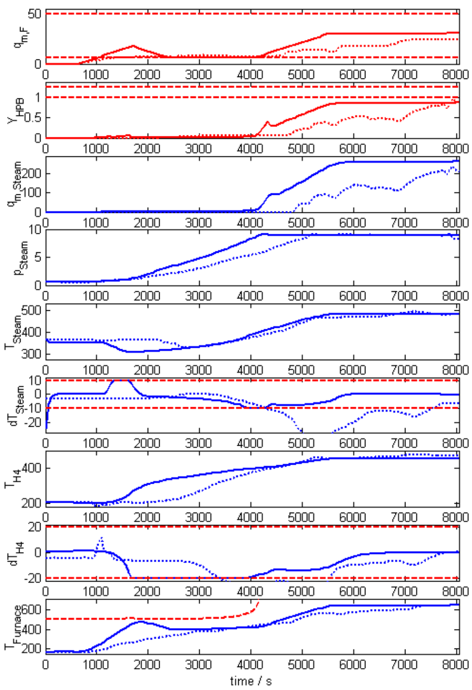
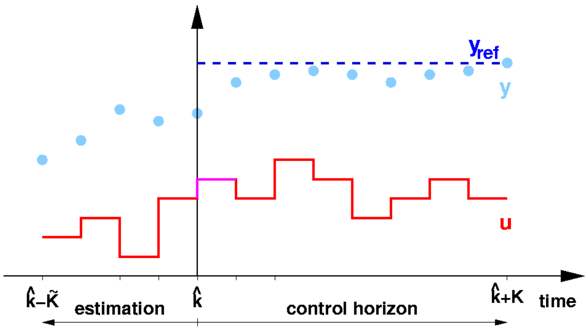
Off-line optimization

- obtain optimal control inputs for given
  - initial states at t0
  - disturbance inputs for [t0,tf]
- piecewise linear parameterization of control inputs
- sampling time interval: 60s
- Can significantly reduce start-up time considering constraints!



Nonlinear Model Predictive Control (NMPC)



Algorithm:

(1) Obtain (estimate) initial states at time ^k

(2) Solve optimal control problem for given prediction horizon K

(3) Apply the first piece of the control trajectory to process

(4) Increase time step ^k and go to (1)

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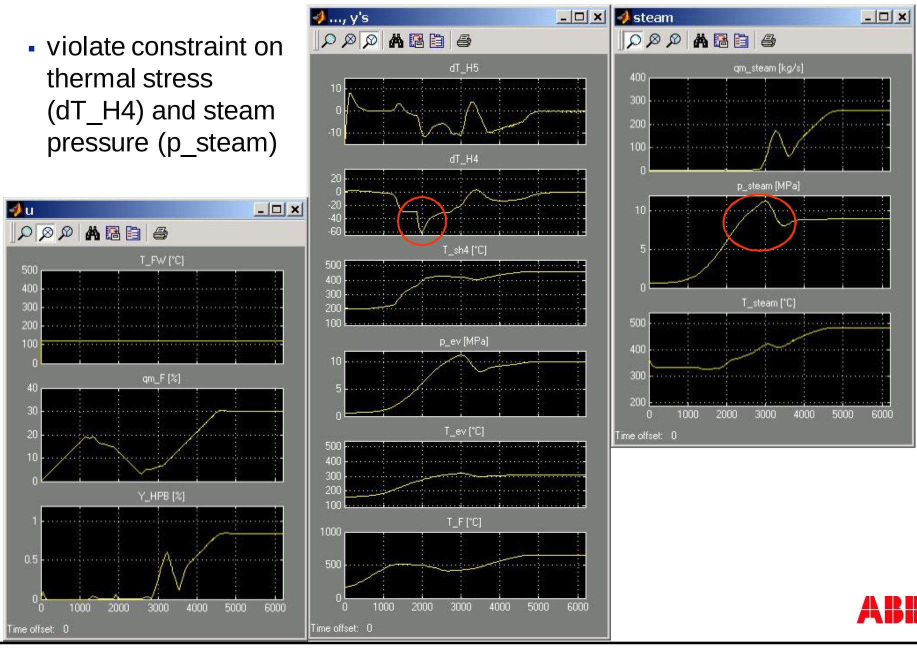
Simulation results for prediction horizon K=10

- violate constraint on thermal stress (dT_H4) and steam pressure (p_steam)



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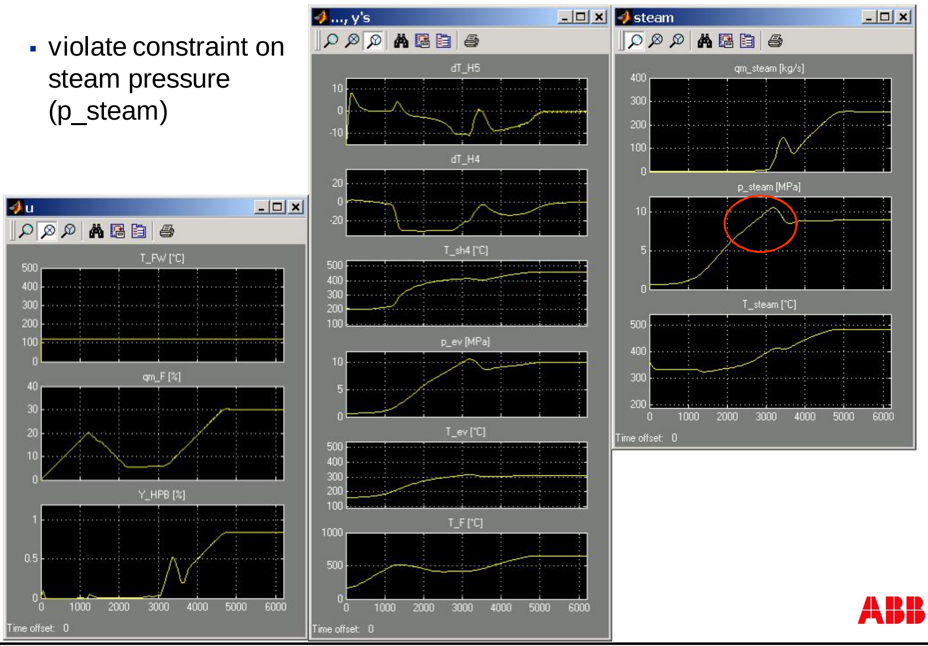
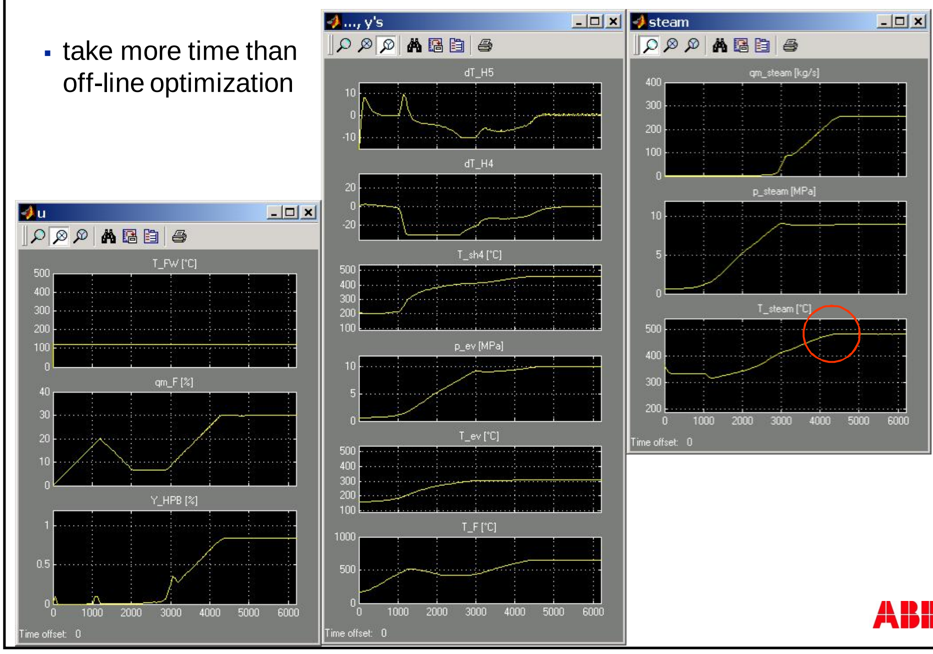
Simulation results for prediction horizon K=20

- violate constraint on steam pressure (p_steam)



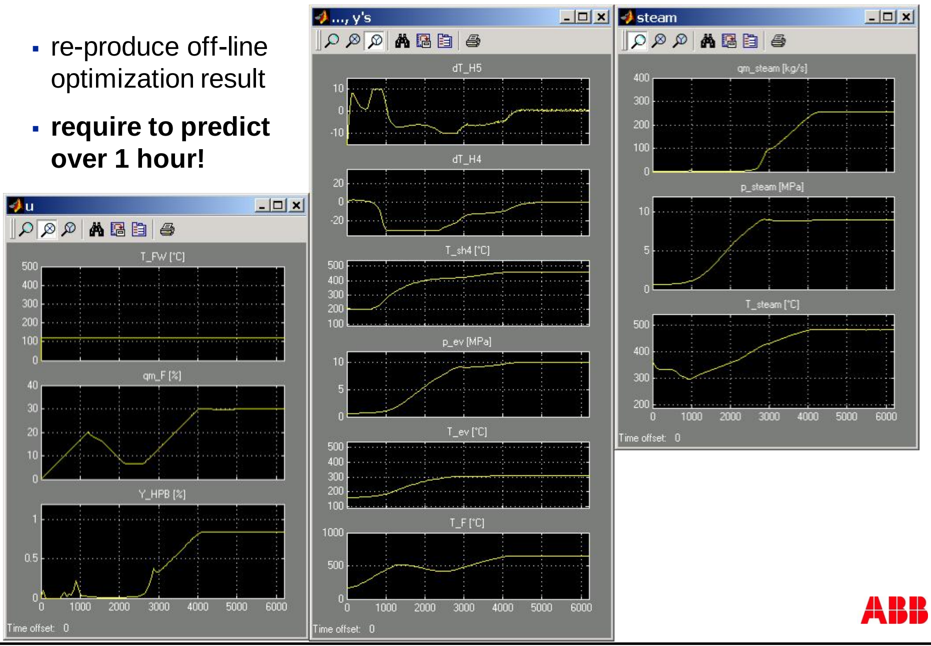
Simulation results for prediction horizon K=30

- take more time than off-line optimization



Simulation results for prediction horizon K=60

- re-produce off-line optimization result
- **require to predict over 1 hour!**



Treatment by HQP optimization solver (dense vs. multi-stage)

- parameterize control $u(t) = u(u^k)$, k=0,1,…,K-1
- convert dynamic optimization problem to discrete-time optimal control problem:

$$J = f_0(x^K) + \sum_{k=0}^{K-1} f_0(x^k, u^k) \longrightarrow \min_{x^0, u^k}$$

$$s.t.$$

$$x^{k+1} = f^k(x^k, u^k), \quad k = 0, \ldots, K-1$$

$$c^k(x^k, u^k) \geq 0, \quad k = 0, \ldots, K-1$$

$$c^K(x^K) \geq 0$$

- solve as large-scale nonlinear programming problem with vector of optimization variables v:

$$\breve{v} = \begin{pmatrix} u^0 \\ u^1 \\ \vdots \\ u^{K-1} \end{pmatrix} \qquad v = \begin{pmatrix} x^0 \\ u^0 \\ x^1 \\ u^1 \\ \vdots \\ x^{K-1} \\ u^{K-1} \\ x^K \end{pmatrix}$$

Problem sizes & CPU times

DAE	ODE		DOCP		Dense CVP		Multi-Stage CVP	
dim(DAE)	n_x	n_u	n_c	K	$\dim(\check{\mathbf{v}})$	CPU times	$\dim(\mathbf{v})$	CPU times
940	11	2	5	10	90	0:44 (00:12)	218	0:20 (0:02)
940	11	2	5	20	160	5:18 (01:21)	418	0:24 (0:03)
940	11	2	5	30	230	13:09 (04:26)	618	0:23 (0:04)
940	11	2	5	60	440	1:34:52 (23:55)	1218	0:45 (0:08)

- DAE: result of model building: high number of DAE variables
- ODE: states (nx) and controls (nu) after compilation using Dymola
- DOCP: soft constraints (nc) and prediction horizon (K)
- Dense CVP: **not applicable** with large K as required for control task
- Multistage CVP: **significantly improved** computational complexity
  -> fulfills real-time requirement: K >= 60, CPU time < 60s

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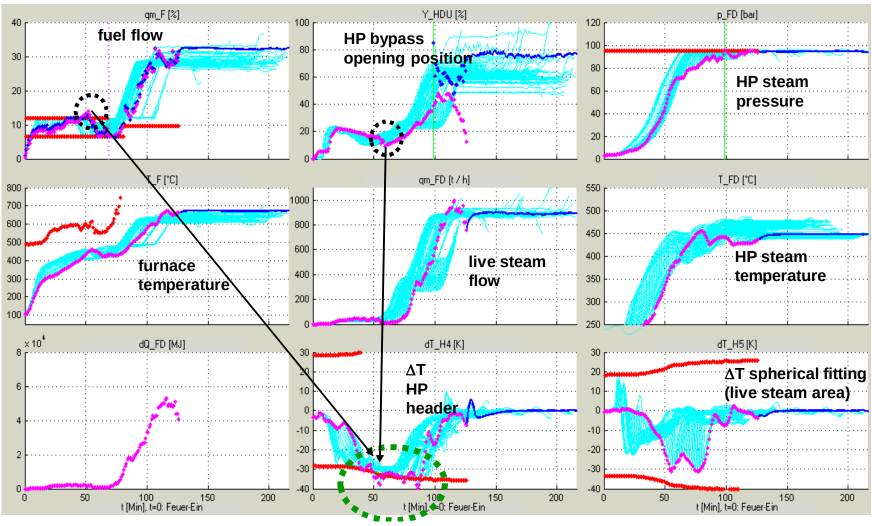
Boiler start-up optimization – NMPC in action



- The ΔT of the critical HP header is controlled along the constraint during 50 min !
- NMPC masters constraint violations, e.g. at time 55: overshoot of fuel flow → ΔT violation → closing of the HP bypass valve

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Conclusions

- Started with evaluation of sparse primal/dual IP algorithm
- Extended to SQP – *It (the SQP solver) can be programmed in an afternoon if one has a quadratic programming subroutine available (Powell, 1978)*
- Most further work went into front-end Omuses and interfaces to simulation models for dynamic optimization
- Many real-world and research applications:
  - Energy management in virtual power plants
  - Anti-sway control of boom cranes
  - Water management in canal systems
  - Start-up of large power plants
  - Predictive control of wind turbines
  - …

HQP is open source, see: https://github.com/omuses/hqp

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References

Ingredients of HQP

- Sparse and/or block structured linear algebra (Meschach: D.E. Steward, Z. Leyk, 1994; E. Arnold, 1987)
- Interior point QP solver (S.J. Wright, 1993, R. Franke, 1994)
- SQP solver (Powell, 1978; K. Schittkowski, 1983)
- Multi-rank Hessian approximation (H.-G. Bock, K.J. Plitt, 1984)
- Block-banded structure, non-condensing (E. Arnold, 1987)

Modeling

- Object-oriented physical systems modeling (Modelica; H.Elmqvist, H.Tummescheit, M.Otter, 2003)
- Integrated Modeling and Optimization (Franke, 1997)

NMPC

- Robust formulation (C.V. Rao, S.J. Wright, J.B. Rawlings, 1998)
- Stability (H. Chen, F. Allgöwer, 1998)
- Real-time feasibility (M. Diehl, 2002)

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