CH 10 BAYESIAN ESTIMATION

SO FAR, IN MLE: MAX PDF OF y GIVEN θ

IN MAXIMUM A POSTERIORI (MAP) ESTIMATION, (A BAYESIAN APPROACH)

IN MAP: MAX. PDF OF θ GIVEN y

USING "BAYES FORMULA" WE GET:

$$\boxed{p(\theta|y) = \frac{p(y|\theta)\, p(\theta)}{p(y)}}$$

$P(A|B) \Longleftrightarrow$ PROB. OF EVENT A GIVEN B

$$f_{MAP}(\theta|y) = \frac{\boxed{f_{ML}(y|\theta)} \cdot \boxed{g(\theta)}}{f_Y(y)}$$

(MAP)

$f_{ML}(y|\theta)$ LIKELIHOOD FCN FROM MLE

$g(\theta)$ A PRIORI KNOWLEDGE

$f_Y(y)$ PDF TO GET MEASUREMENT y (INDEP. OF θ)

$$\hat{\theta}_{MAP}(y) = \arg\max_{\theta} f_{MAP}(\theta|y)$$

$$= \arg\min_{\theta} \left(-\log f_{MAP}(\theta|y)\right)$$

$$= \arg\min_{\theta} \left\{ \left(-\log f_{ML}(y|\theta)\right) + \left(-\log g(\theta)\right) - \cancel{\left(-\log f_Y(y)\right)} \right\}$$

EXAMPLE:

A PRIORI: $\Theta \sim \mathcal{N}(\bar{\Theta}, \Sigma_\Theta)$

$$\boxed{g(\Theta)} = \text{const.} \quad \exp\left(-\frac{1}{2}(\Theta-\bar{\Theta})^T \Sigma^{-1} (\Theta-\bar{\Theta})\right)$$

$$-\log g(\Theta) = \frac{1}{2}(\Theta-\bar{\Theta})^T \Sigma_\Theta^{-1} (\Theta-\bar{\Theta})$$

REGULARIZATION TERM $-\log g(\Theta)$ IS ADDED TO ML-OBJECTIVE, IT CAN INTRODUCE BIAS. CAN INTERPRET AS

"PSEUDO MEASUREMENT" OF Θ WITH VALUE $\bar{\Theta}$ AND ERROR Σ_Θ

EX 2

REINTERPRETATION OF "TRAJECTORY FITTING"

$$\boxed{y(k) = a(1) \cdot y(k-1) + a(2)\, y(k-2) + b(1)\, u(k-1) + \varepsilon_e(k)}$$

MEASUREMENTS : $y^{MEAS}(k) = y(k) + \varepsilon_y(k)$

$u^{MEAS}(k) = u(k) + \varepsilon_u(k)$

ASSUMED $\varepsilon_e(k) \sim \mathcal{N}(0, \sigma_e^2)$

$\varepsilon_y(k) \sim \mathcal{N}(0, \sigma_y^2)$

$\varepsilon_u(k) \sim \mathcal{N}(0, \sigma_u^2)$

$$\Theta = \begin{bmatrix} a(1) \\ a(2) \\ b(1) \\ u(1) \\ \vdots \\ u(N) \\ y(1) \\ \vdots \\ y(N) \end{bmatrix}$$

WE FOUND $\hat{\Theta}$ BY

$$\hat{\Theta} = \arg\min_{\Theta} \sum_i \left\{ \left(\frac{y^{MEAS} - y}{\sigma_y} \right)^2 + \left(\frac{u^{MEAS} - u}{\sigma_u} \right)^2 + \underline{\underline{\left(\frac{y(k) - a(1) \cdot y(k-1) \ldots}{\sigma_e} \right)^2}} \right\}$$

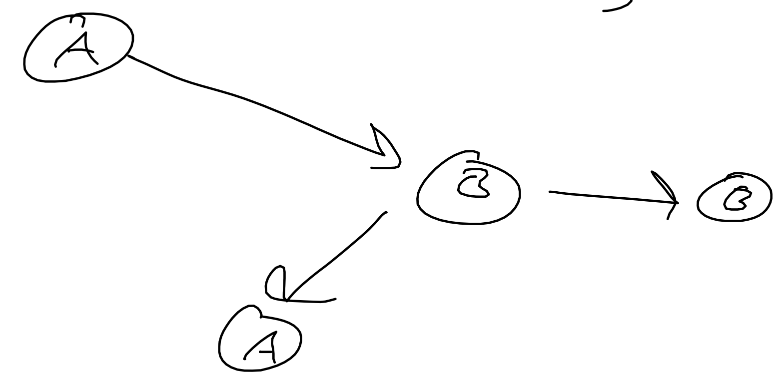
A PRIORI KNOWLEDGE

BRAINSTORM FOR EXAMPLES

A • HEATING WATER ON A PLATE 7

B • COLLIDING BALLS 5

(ELASTICITY, ...)



C • CRANE

(INPUT: SPEED OF MOTOR,
OUTPUT: MASS) 4

(LENGTH OF ROPE)

D • HOT AIR BALLON 10
INPUT : HEATING (MEASURE)
OUTPUT : ALTITUDE (")

→ MODELLING

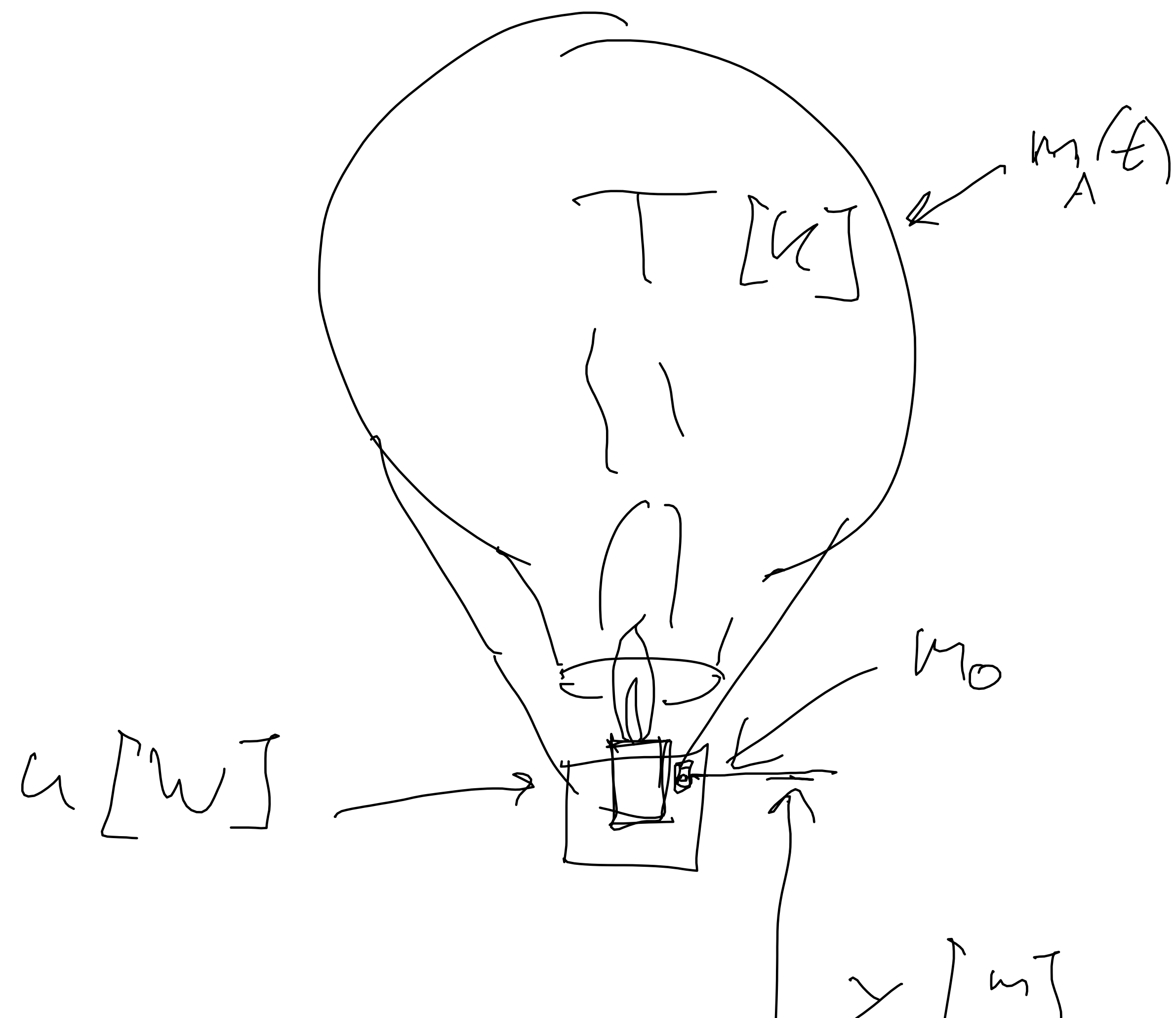
↓

E • ANIMAL POPULATION
MONKEYS
&
SNAKES 6

EXAMPLE: HOT AIR BALLOON

INPUTS: HEATING u

OUTPUTS: ALTITUDE y



CHOICES TO MAKE

- BLACK OR WHITE BOX?
- CONT. VS. DISCR.

DECISIONS

- ONLY VERTICAL MOTION
- OUTSIDE TEMP. CONST.
- PRESSURE DEP. ON ALTITUDE
- NEGLECT AIR FRICTION
- NEGLECT COOLING VIA INCOMING AIR WHEN BALLOON COOLS
- NEGLECT VARYING HEAT CAPACITY

$$\boxed{\underline{m}\ddot{y} = -\underline{m}g + F_{BUOYANCY}}$$

PARAMETERS

- m MASS OF WHOLE APPARATUS
- T TEMPERATURE IN BALLOON

$$\boxed{F_{BUOYANCY} = ?}$$

$m \sim T$

$\dot{T} = ?$

T_0

$$\left[pV = \underline{n} k_B T \right]$$

$$\boxed{\texttt{ode45}}$$

$$p(y) V = T(t) \cdot m_A(t) \cdot \underline{\underline{k}}$$

$$\Updownarrow$$

$$m_A(t) = \left[\frac{V}{k}\right] \cdot \frac{p(y)}{T(t)}$$

$p(y)$: KNOWN

$$F_{BUOYANCY}(y) = g \cdot m_{OUTSIDE\text{-}AIR}(y)$$

$$m_{OUTSIDE\text{-}AIR}(y) = \left[\frac{V}{k}\right] \frac{p(y)}{T_0}$$

$m(T)$ $\boxed{m(t) = m_0 + m_A(t) = m_0 + \left[\frac{V}{k}\right] \frac{p(y(t))}{T(t)}}$

STATE: $\begin{bmatrix} y \\ \dot{y} \\ T \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\left(C \cdot \dot{T} = u - k_2 (T - T_0) \right)$$

$$\begin{aligned} \dot{y} &= \dot{y} \\ \ddot{y} &= -g + \frac{F_B(y)}{m(T)} \\ \dot{T} &= \frac{u}{C} - \frac{k_2}{C} T + \frac{k_2}{C} T_0 \end{aligned}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g + \dfrac{F_B(x_1)}{m(T)} \end{cases}$$

$$\boxed{\dot{x}_3 = \frac{u}{C} - \frac{k_2}{C} x_3 + \frac{k_2}{C} T_0}$$

$$\boxed{\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x, u) \end{aligned}}$$