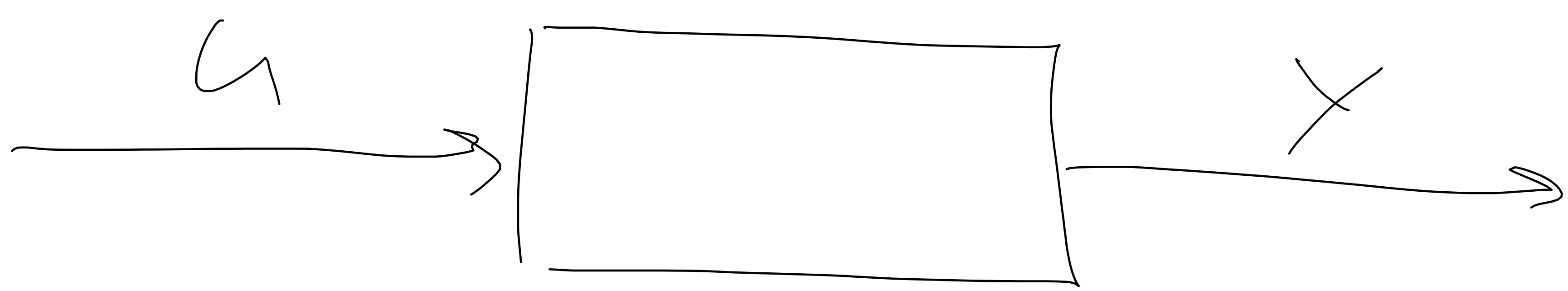
CH 6 [CONT]

DYNAMIC MODELS



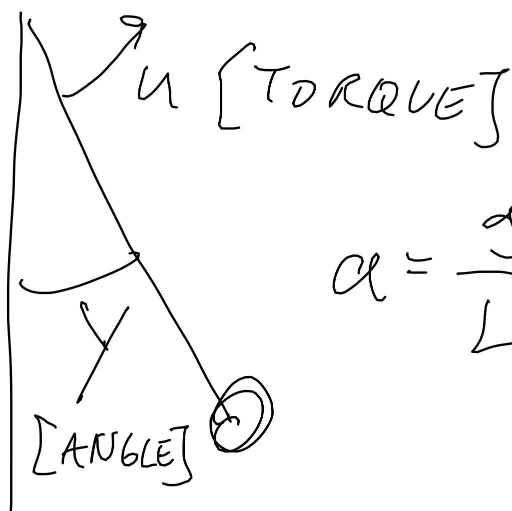
DIVISION LINES

a) $\boxed{\text{NONLINEAR}}$ VS $\boxed{\text{LINEAR}}$

E.G. $\ddot{y}(t) = -a \cdot \sin(y(t)) + b \cdot u(t)$

$\ddot{y} = -a\, y + b u$

["DOUBLE INPUT DOUBLE OUTPUT"]

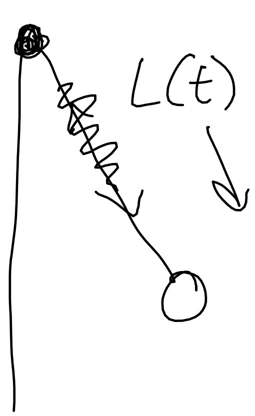


$a = \frac{g}{L}$

b) $\boxed{\text{TIME VARYING}}$ VS $\boxed{\text{TIME INVARIANT}}$

E.G. $\boxed{\ddot{y}(t) = -\underline{a(t)} \cdot \sin(y(t)) + b \cdot u(t)}$

ABOVE IN a) (BOTH)



$a(t) = \frac{g}{L(t)}$

["TODAY EQUALS TOMORROW"]

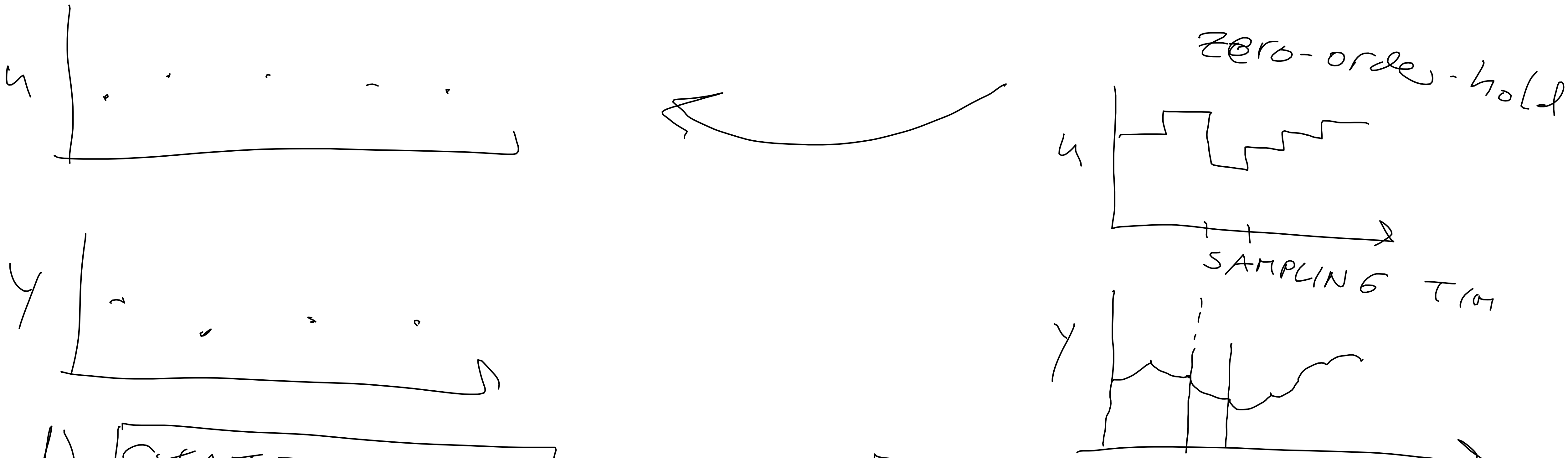
SPECIAL CASE:

LINEAR TIME INVARIANT (LTI)

c) $\boxed{\text{DISCRETE TIME}}$ VS. $\boxed{\text{CONTINUOUS TIME}}$

E.G. $y(k) = a_1 y(k-1) + b_1 u(k-1)$

$\dot{y}(t) = a_1 y(t) + b_1 u(t)$



d) $\boxed{\text{STATE SPACE}}$ VS $\boxed{\text{INPUT-OUTPUT-MODELS}}$

E.G.

$$x(k+1) = A \cdot x(k) + B u(k)$$

[FIRST ORDER!]

$$y(k) = C \cdot x(k)$$

[DISCR. TIME LTI]

ONLY $y(k), y(k-1), \dots$

AND $u(k), \dots$

NO $x(k)$!

HIGHER ORDER ALLOWED

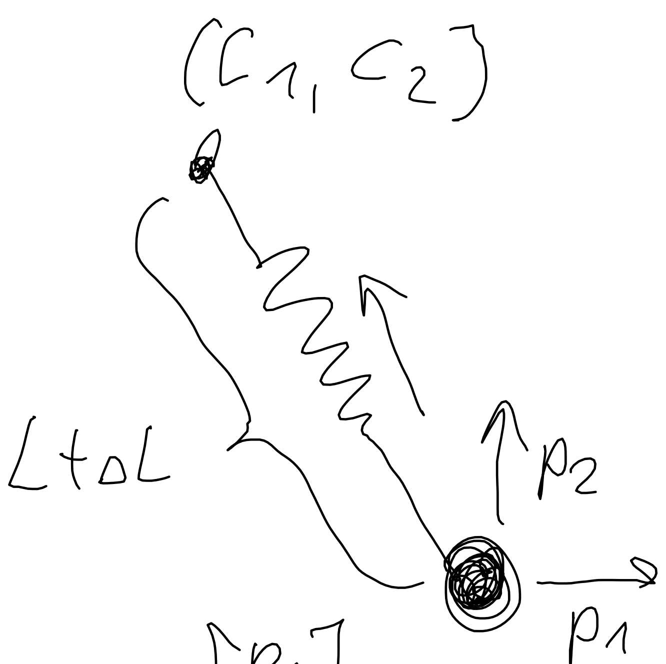
E.G. $y(k) = a_1 y(k-1) + a_2 y(k-2)$

DEF: MODEL ORDER

i) STATE SPACE: DIMENSION OF $x(k)$ "STATE DIMENSION"

ii) I/O-MODELS, DISCRETE: HOW MANY PAST SAMPLES OF y WE NEED TO PREDICT NEXT OUTPUT

iii) I/O, CONT. TIME: HIGHEST TIME DERIVATIVE OF y



$$m \cdot \ddot{p} = F$$

$$m \cdot \dot{v} = F$$

$$F = \frac{(c-p)}{\|c-p\|_2} \cdot \left(\|c-p\|_2 - L\right) \cdot K + \begin{bmatrix} 0 \\ -m \cdot g \end{bmatrix}$$

$$p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix}$$

$$\dot{v} = \frac{F}{m} = (c-p) \cdot \left(1 - \frac{L}{\|c-p\|_2}\right) \cdot \frac{K}{m} + \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

$$\dot{p} = v$$

$$x = \begin{bmatrix} p_1 \\ p_2 \\ v_1 \\ v_2 \end{bmatrix}, \quad f(x) = \begin{bmatrix} v_1 \\ v_2 \\ (c_1 - p_1)\left(1 - \frac{L}{\|\cdot\|}\right)\frac{K}{m} \\ (c_2 - p_2)\left(\;\text{"}\; \right)\frac{K}{m} - g \end{bmatrix}$$

$$\dot{x} = Ax + b$$

NL, TI, CT, SS

$$\boxed{\dot{x} = f(x)}$$

$$\frac{L}{\|c-p\|_2} = \frac{L}{\sqrt{(c_1-p_1)^2 + (c_2-p_2)^2}}$$