Hello world!

CH 5 [CONT]

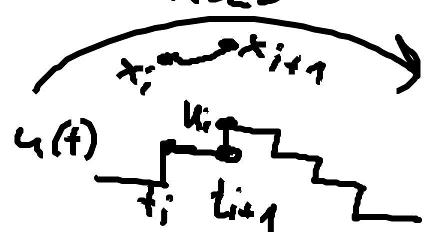
LTI

CAN TRANSFORM MODEL TYPES INTO EACH OTHER

E.G. FROM CONT. TO DISCRETE TIME (STATE SPACE)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

ZERO ORDER HOLD



$$t_{i+1} = t_i + \Delta t$$

$$x(t_{i+1}) = A_d\, x(t_i) + B_d\, u_i$$
$$y(t_i) = C x(t_i) + D u_i$$

$$A_d = \exp(A \Delta t)$$
$$B_d = \int_0^{\Delta t} \exp(A\tau) \cdot B \, d\tau$$

$$G(s) = \frac{Y(s)}{U(s)} = C\cdot(sI-A)^{-1}B + D = \frac{P_{NOM}(s)}{P_{DEN}(s)}$$

$$s\cdot X(s) = A\,X(s) + B\,U(s) \iff (sI-A)X = BU$$

$$\Updownarrow$$

$$Y(s) = C\cdot X(s) + D\,U(s) \qquad X = (sI-A)^{-1}BU$$

$$\frac{p(s)\,\cancel{(s+a)}}{q(s)\,\cancel{(s+a+\varepsilon)}}$$

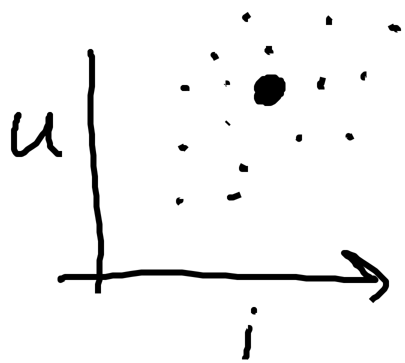
$\underline{|G(j\omega)|}_{dB}$

$$y(k) = -a_1\,y(k-1) + \dots + u_1\,u(k-1)$$

CH 7 MAXIMUM LIKELIHOOD ESTIMATION

[LJUNG, CH 7.3]

BACK TO RESISTOR:



KNOW: $R = \frac{U_{TRUE}}{i_{TRUE}}$

TRUTH: Θ (UNKNOWN)

WE HAVE: Y_N (MEASURED, WITH ERRORS!)

PDF: $\underline{\underline{p(Y_N|\Theta)}}$ PDF OF Y_N GIVEN Θ

WE KNOW THIS FUNCTION, BUT NOT Θ

DEF: $f(\theta) := p(y_N|\theta)$ IS CALLED "LIKELIHOOD FUNCTION"

MAXIMUM LIKELIHOOD ESTIMATOR

$$\hat{\theta}_N = \arg\max_{\theta} \; p(y_N|\theta)$$

$$= \arg\max_{\theta} \; \log p(y_N|\theta)$$

$$= \arg\min_{\theta} \left(-\log p(y_N|\theta)\right)$$

"NEGATIVE LOG-LIKELIHOOD"

RESISTOR EXAMPLE:

$$\theta_{TRUE} = (u_{TRUE}, i_{TRUE})$$

PDF OF ONE MEASUREMENT:

$$p(u_{meas} | \bar{u}) = \frac{1}{\sqrt{2\pi}\, \sigma_u} \cdot \exp\left(-\frac{(u_{meas} - \bar{u})^2}{2\sigma_u^2}\right)$$

(ASSUME WE KNOW σ_u)

(SAME FOR i)

$$p(u_N^{MEAS} | \bar{u}) = \prod_{k=1}^{N} p(u_k^{meas} | \bar{u}) = \prod \exp\left(-\frac{(u_k^{meas} - \bar{u})^2}{2\sigma_u^2}\right)$$

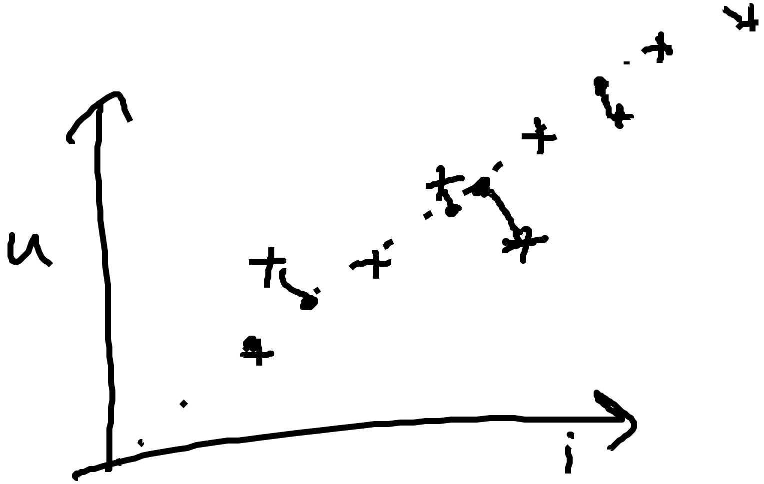
$\cdot$ CONST

1st AIM: ESTIMATE u_{TRUE}

$$\hat{u}_N = \arg\min_{\bar{u}} \left(-\log p(u_N^{MEAS} | \bar{u})\right)$$

$$= \arg\min_{\bar{u}} \sum_{k=1}^{N} \frac{(u_k^{MEAS} - \bar{u})^2}{2\sigma_u^2} + \text{CONST} = \frac{1}{N} \sum u_k^{MEAS}$$

NEW RESISTOR EXAMPLE



[DIFFERENT CURRENTS $i(k)$

SAME RESISTOR:

$$\boxed{u(k) = R \cdot i(k)}$$

$$\boxed{\begin{aligned} u^m(k) &= u(k) + \varepsilon_u(k) \\ i^m(k) &= i(k) + \varepsilon_i(k) \end{aligned}}$$

MEASUREMENTS: $[u^m(k), \; i^m(k)] = y_N$

KNOWN ERRORS: $\sigma_u(k), \; \sigma_i(k)$

$\Theta = [\underline{R}, i(1), \ldots, i(N)]^T$ (GET $u(k) = R \cdot i(k)$)

LIKELIHOOD TO GET $u^m(k)$: $p(u^m(k)/\theta) = \frac{1}{\sqrt{\cdots}} \cdot \exp\left(-\frac{(u^m(k) - R i(k))^2}{2\sigma_u(k)^2}\right)$

NEG-LOGLIKELIHOOD FOR y_N, Θ:

$$-\log(p(y_N|\theta)) = \sum_{k=1}^{N} \frac{(u^m(k) - R \cdot i(k))^2}{2\sigma_u(k)^2} + \sum_{k=1}^{N} \frac{(i^m(k) - i(k))^2}{2\sigma_i(k)^2}$$

SIMILER NEW RESISTOR EXAMPLE:

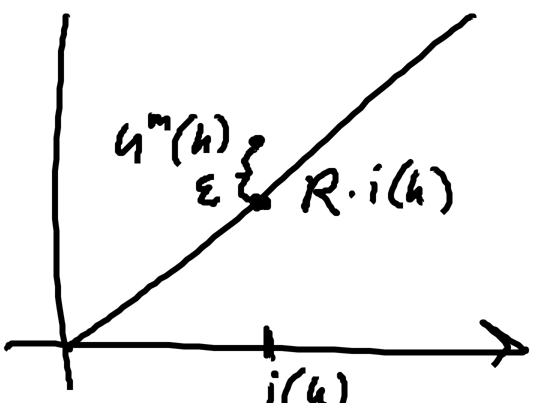
$i(k)$ KNOWN EXACTLY

$$\boxed{u^m(k) = R \cdot i(k) + \varepsilon_u(k)} \qquad \theta = R$$

GIVEN $y_N = [u^m(1), \dots, u^m(N)]^T$

$$\varepsilon_u(k) \sim \mathcal{N}(0, \sigma_u(k)^2)$$

$$-\log p(y_N|\theta) = \sum_{k=1}^{N} \frac{(\overbrace{u^m(k) - R\, i(k)}^{\varepsilon_u(k)})^2}{2\sigma_u(k)^2}$$



$$= \frac{1}{2} \| b - A \cdot R \|_2^2 \qquad b = \begin{bmatrix} \frac{u^m(1)}{\sigma_u(1)} \\ \vdots \end{bmatrix}, \; A = \begin{bmatrix} \frac{i(1)}{\sigma_u(1)} \\ \vdots \end{bmatrix}$$

LINEAR LEAST SQUARES

MLE FOR LTI-SYSTEMS

LLS-REVISITED:

$$\underline{\underline{y}}(k) = -a_1 \underline{\underline{y}}(k-1) - a_2 \underline{\underline{y}}(k-2) + b_1 \underline{\underline{u}}(k-1) + \varepsilon(k)$$

EQUATION ERROR

MORE REALISTIC:

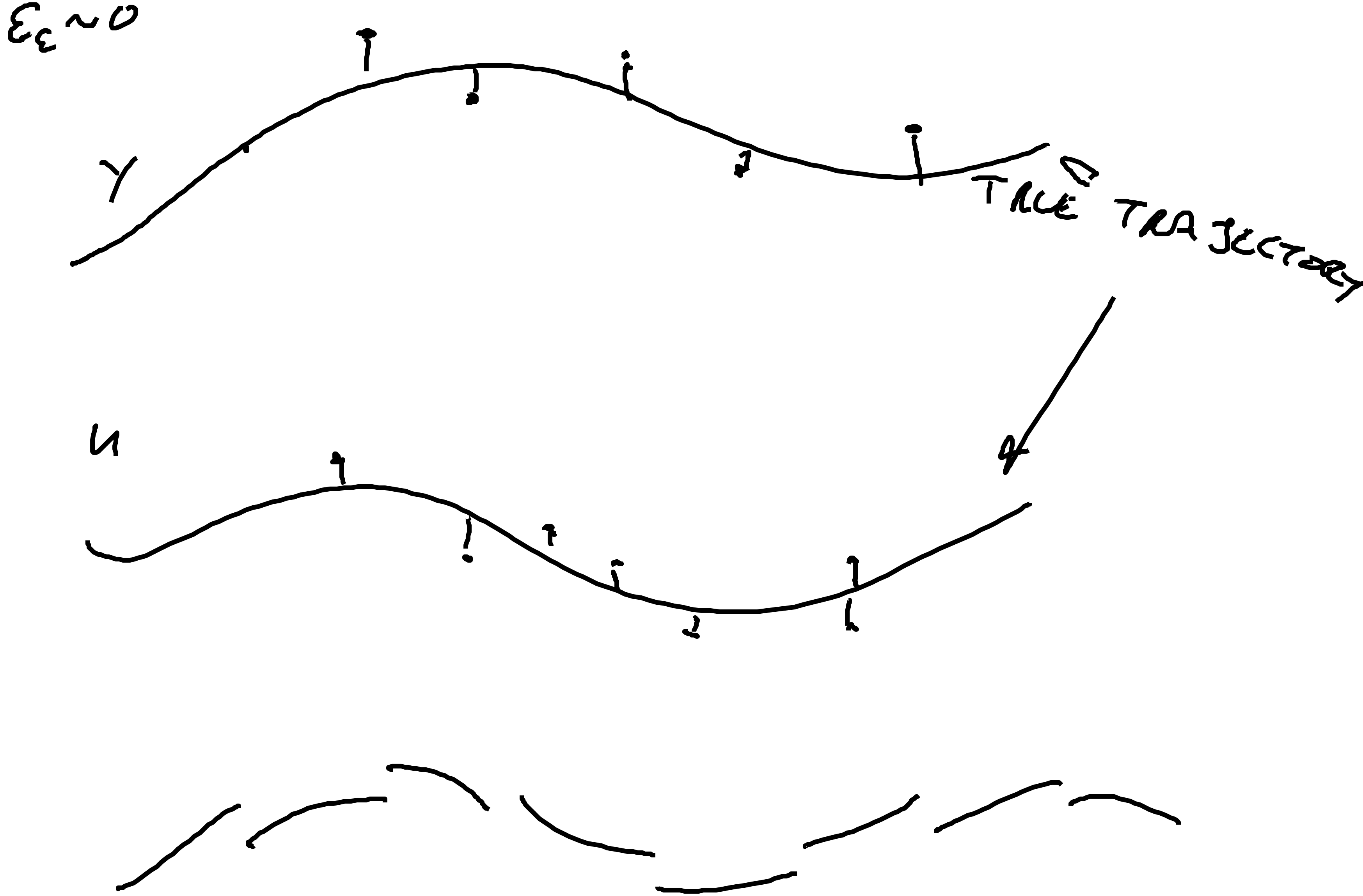
$$y^m(k) = y(k) + \varepsilon_y(k)$$

$$u^m(k) = u(k) + \varepsilon_u(k)$$

$$\eta_N = (y^m(1), \dots, u^m(1), \dots)^T$$

$$\theta = [a_1 \; a_2 \; b_1 \; u(1) \; u(2) \dots y(1) \dots]^T$$

$$-\log p(\eta_N | \theta)$$


$\varepsilon_\varepsilon \sim 0$
Y
TRUE TRAJECTORY
U

$$\text{lsqnonlin}(\text{residual}, \theta_0) = \arg\min_{\theta} \|F(\theta)\|_2^2$$

$$\text{residual} = \begin{bmatrix} \varepsilon(1) \\ \vdots \\ \vdots \\ \\ \varepsilon(N) \end{bmatrix} = F(\theta)$$

FOR LLS:

$$F(\theta) = Y_N - \Phi_N \cdot \theta$$

$F(\theta)$ TYPICALLY NONLINEAR IN PARAMETERS

MODEL PHYSICALLY

GENERATE $\boxed{\text{PSEUDO-DATA}}$

EQUATION ERROR (PEM)

LLS

MLE

NLS