Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2014

Exercise 6: Nonlinear Least Squares

(to be returned on Dec 2, 2014, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

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Please remember to provide a solution on paper (written or typed) including all the necessary graphs from MATLAB. The MATLAB code (.m-files) should be sent to
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Aim of this sheet is to formulate and solve a maximum-likelihood problem with nonlinear least squares using MATLAB, using the MATLAB command `lsqnonlin`. As a central model, we are using the one-dimensional race car from Exercise 5:

$$\dot{v}_X(t) = C_1 D(t) - C_2 - C_3 v_X(t),$$

with as input the dutycycle D [-]. Our aim is to estimate the three unknown parameters C_1 [ms^{-2}] the motor constant, and C_2 [ms^{-2}], C_3 [s^{-1}] the zeroth and first order friction constants respectively. Because we do not know them, we will also have to estimate the initial conditions $p_X(0), v_X(0)$.

Exercise Tasks

1. Write in MATLAB a simulation function `simstep` that takes the following seven real numbers $C_1, C_2, C_3, p_X(0), v_X(0), \Delta T, D$ as inputs, and computes from them the state $p_X(T), v_X(T)$ at the time ΔT, assuming a constant value D on the interval $[0, \Delta T]$. Hint: use the solution formula from Exercise Sheet 5.

2. Write a MATLAB simulation loop `simloop` around `simstep` that simulates N time steps of length ΔT and takes as input, besides $C_1, C_2, C_3, p_X(0), v_X(0), \Delta T$ and N a vector of values D_k, $k = 1, \ldots, N$ that are assumed piecewise constant on each interval $[(k-1)\Delta T, k\Delta T]$. As output, the function should generate the values $p_X(k\Delta T)$ and $v_X(k\Delta T)$ for $k = 0, 1, \ldots, N$. Test your simulation loop with some values for $C_1, C_2, C_3, p_X(0), v_X(0), \Delta T, N$ and constant D for all intervals. Plot the trajectory. (3 points)

3. Load `data6_1.txt`. These are time-dependent measurements of the form `|time|velocity|D|`. We assume no noise on the measurements of time and on D, and the velocity measurements i.i.d. measurement errors.

   (a) First, we estimate $[C_2, C_3, v_X(0)]$ simultaneously. Assume that $C_1 = 10$ is known. First formulate a residual function `[res]=residual(vel, theta)` that computes the misfit $M(\theta) - y$ between the model predictions and the actual measurements. Then compute the nonlinear least squares fit of the velocity using the MATLAB command `lsqnonlin`. Hint: you can call a MATLAB script to load the data from the `residual` function.

   Plot the simulated versus the measured velocity values. What is the maximum likelihood estimate for $[C_2, C_3]$? Assume Gaussian additive noise on the measurements. (3 points)

   (b) * Estimate the confidence ellipsoid around the estimate of $[C_2, C_3, v_X(0)]$, using the same strategy as for linear least squares in Exercise Sheet 4, but replacing the matrix Φ_N by the Jacobian $\frac{\partial M}{\partial \theta}$. Hint: You can compute this Jacobian from the ODE solution for the velocity (Exercise 5). (3 bonus points)

   (c) * Try to estimate the four parameters $[C_1, C_2, C_3, v_X(0)]$ simultaneously. What values do you get? Do these sound reasonable? If not, try to find an explanation why the estimation failed. Hint: Estimate the confidence ellipsoid. (2 bonus points)

4. Load `data6_2.txt`. These are time-dependent measurements of the form `|time|velocity|D|`. Now, the dutycycle is not constant over the given interval.

   (a) Estimate the four parameters $[C_1, C_2, C_3, v_X(0)]$ using the new data and the same procedure as before. (3 points)

   (b) * Estimate the confidence ellipsoid for the estimate. (2 bonus points)

This sheet gives in total 10 points and 7 bonus points