$$\dot{x} = f(x,u)$$

Differential Equation Types

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Overview

- Ordinary Differential Equations (ODE)
- Differential Algebraic Equations (DAE)
- Partial Differential Equations (PDE)
- Delay Differential Equations (DDE)

Comments

Ordinary Differential Equations (ODE)

- General ODE:

$$\dot{x}(t) = f(x(t), u(t), \epsilon(t), p)$$

- states $x(t)$, control inputs $u(t)$, disturbances $\epsilon(t)$, unknown parameters p (constant in time). All variables are vector valued.
- Here, $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial x}{\partial t}$. Total and partial derivative coincide as x only depends on t.
- for notational simplicity, we usually omit time dependence and write $\dot{x} = f(x, u, \epsilon, p)$
- for even simpler notation, from now on we omit $u(t), \epsilon(t)$ and p in this talk. They should be added again when necessary.
- Standard form of ODE for this talk:

$$\dot{x} = f(x)$$

Comments

Examples (what are their state vectors?)

$$\dot{x} = \begin{bmatrix} x_2 \\ -\frac{g}{L} \cdot \sin x_1 \end{bmatrix}$$

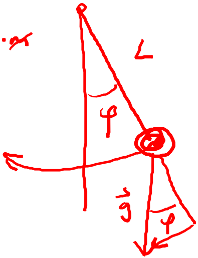
f

$$x = \begin{bmatrix} \varphi \\ \omega \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$L m \dot{\omega} = -\sin\varphi \cdot g \cdot m$$

$$\dot{\varphi} = \omega$$

$$\begin{bmatrix} \dot{\omega} \\ \dot{\varphi} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} -\sin\varphi \, \frac{g}{L} \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{g}{L} \cdot \sin x_1 \\ x_2 \end{bmatrix}$$



$$f(x) = \begin{bmatrix} x_2 \\ -\frac{g}{L} \cdot \sin x_1 \end{bmatrix}$$

- Pendulum
- Hot plate with pot
- Continuously Stirred Tank Reactors (CSTR)
- Robot arms
- Moving robots
- Race cars
- Airplanes in free flight

Comments

Comments

Differential Algebraic Equations (DAE)

- similar to ODE
- besides *differential states* $x \in \mathbb{R}^{n_x}$ there are also *algebraic states* $z \in \mathbb{R}^{n_z}$
- Standard form of DAE ("semi-explicit DAE"):

$$\dot{x} = f(x, z)$$
$$0 = g(x, z)$$

- The *algebraic equations* $g(x, z) = 0$ implicitly determine z
- we usually have to assume that the Jacobian $\frac{\partial g}{\partial z} \in \mathbb{R}^{n_z \times n_z}$ is invertible ("index one")

Comments

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